Homework 10

1. Perform a local analysis of the algebraic equation \( y = e^{xy} \) near \( x = 1/e \) by substituting \( y = e + \delta(x) \), where \( \delta \to 0 \) as \( x \to 1/e \). Solve approximately for \( \delta(x) \) to show that near \( x = 1/e \), \( y(x) \) has a square-root singularity.

2. Does the solution to the initial-value problem \( y'(x) = \sqrt{x^2 + y^2} \ [y(0) = a] \) remain finite for all \( x \)?

3. Show that the leading behavior of an explosive singularity of the Thomas-Fermi equation \( y'' = y^{3/2}x^{-1/2} \) is correctly given by

\[
y(x) \sim \frac{400a}{(x - a)^3}, \quad x \to a.
\]

4. Let \( yd^4y/dx^4 = 1 \ [y(0) = y''(0) = y(1) = y''(1) = 0] \). Find the asymptotic behavior of \( y(x) \) as \( x \to 0+ \). Try several terms involving combinations of logs and powers.

5. Find the leading asymptotic behavior of the solution to \( yy'' = x^3y'^2 \) as \( x \to +\infty \).