Chapter 1, Section 1.1

Problems 1-2 are routine verifications by direct substitution of the suggested solutions into the given differential equations.

3.

\[ y_1 = \cos 2x \]

so

\[ y_1' = -2 \sin 2x, \quad y_1'' = -4 \cos 2x \]

Plugging \( y_1 \) and \( y_1'' \) into the DE yields

\[ -4 \cos 2x + 4 \cos 2x = 0 \]

which is an identity. So \( y_1 \) is a solution of the DE

\[ y'' + 4y = 0 \]

.

\[ y_2 = \sin 2x \]

so

\[ y_2' = 2 \cos 2x, \quad y_2'' = -4 \sin 2x \]

Plugging \( y_2 \) and \( y_2'' \) into the DE yields

\[ -4 \sin 2x + 4 \sin 2x = 0 \]

which is an identity. So \( y_2 \) is a solution of the DE

\[ y'' + 4y = 0 \]

.

7. Solution: If \( y_1 = e^x \cos x \), then \( y_1' = e^x(\cos x - \sin x) \) and \( y_1'' = -2e^x \sin x \). Plugging \( y_1 \) and its derivatives into the DE yields

\[ -2e^x \sin x - 2e^x(\cos x - \sin x) + 2e^x \cos x = 0 \]

which is simplified into

\[ e^x(-2 \sin x - 2 \cos x + 2 \sin x + 2 \cos x) = 0 \]
which is an identity. So $y_1$ is a solution of the DE.

If $y_1 = e^x \sin x$, then $y_1' = e^x(\sin x + \cos x)$ and $y_1'' = 2e^x \cos x$. Plugging $y_2$ and its derivatives into the DE yields

$$2e^x \cos x - 2e^x(\sin x + \cos x) + 2e^x \sin x = 0$$

which is an identity. So $y_2$ is a solution of the DE.

14 Solution: Plugging $y = e^{rx}$ into the DE yields

$$4r^2 e^{rx} = e^{rx}$$

that simplifies into

$$4r^2 = 1$$

Thus solving this quadratic equation gives

$$r = \pm \frac{1}{2}$$

15 hint: $r^2 + r - 2 = 0$ gives the roots $r = -2$ or $r = 1$.
16 hint: $3r^2 + 3r - 4 = 0$ gives the roots

$$r = \frac{-3 \pm \sqrt{57}}{6}$$

Remind: The roots of the quadratic equation

$$ax^2 + bx + c = 0$$

where $a, b, c$ are constants are given by the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

provided that $b^2 - 4ac \geq 0$.

35 Solution:

$$\frac{dN}{dt} = k(P - N)$$

where $k$ is a constant.

In this problem, $N(t)$ denotes the people that who have heard the rumor, $P$ is the fixed population and $P - N$ is the people who have not heard the rumor. The time rate of the change of $N$ is translated into the first derivative of $N$.

36 Solution:

$$\frac{dN}{dt} = kN(P - N)$$

where $k$ is a constant.