1(20 points): In each of the following five problems, the roots of the characteristic equation of a certain homogenous differential equation are given. Write the general solution of the homogeneous differential equation.

- **Roots:** -1/5, -1/5, -1/5, 0, $\sqrt{2}$
  General solution: ____________________________

- **Roots:** 3i, -3i, 2+4i, 2-4i
  General solution: ____________________________

- **Roots:** 2+i, 2+i, 2-i, 2-i
  General solution: ____________________________

- **Roots:** -2,5,5,-1+i,-1-i
  General solution: ____________________________

- **Roots:** -2,0,0,3,$\sqrt{7}i$, $-\sqrt{7}i$
  General solution: ____________________________

2. (10 points) Given the following nonhomogeneous equation:

$$y^{(4)} + 25y'' = f(x)$$

(a) Find a general solution $y_c$ of the associated homogeneous equation.

(b) For each of the following three $f(x)$, set up the appropriate form of a particular solution $y_p$, but do NOT determine the value of the coefficients.

  (i) $f(x) = x^2 \sin 5x$;  (ii) $f(x) = x^2 e^{5x}$;  (iii) $f(x) = x^2 + \sin 3x$

3(10 points): Solve the following initial value problem:

$$y'' + 3y' + 2y = 12e^x; \quad y(0) = -3, \quad y'(0) = 3.$$
5. **(15 points)**: Transform the given system into an equivalent system of first-order differential equations.

\[ x'' = 5x - 4y + 3 \sin 2t, \quad y'' = -4x + 5y. \]

6. **(15 points)** Find general solutions of the given linear system by using the systematic elimination procedure.

\[ x' = 2x - 3y + 2 \sin 2t, \quad y' = x - 2y - \cos t \]

7. Use the Wronskian to prove the the functions

\[ f(x) = x^2, \quad g(x) = xe^x, \quad h(x) = x^2e^x \]

are linearly independent on the real line.