第一届北京大学计算数学研究生论坛

论　坛　简　介

北京大学计算数学研究生论坛一个由北京大学数学科学学院科学与工程计算系发起并主办，学生筹办的研究生学术交流活动。

本论坛的宗旨是

- 促进北京大学计算数学方向研究生之间的学术交流，提供一个广泛的交流思想、增进友谊的平台；
- 加强相互之间的沟通与交流，促进计算数学方向研究生在学术氛围与创新能力等方面的发展；
- 通过论坛活动，带动计算数学方向研究生的研究热情。

本次活动将包含 11 个学术报告，欢迎校内外相关领域的师生参与并给予指导。该论坛将为注册人员提供免费的午餐和晚餐。

论坛信息：

举办时间： 2013 年 11 月 16 号
举办地点： 北京大学理科一号楼 1560

论坛筹备委员会：(按拼音字母顺序)

指导老师： 李　若教授、汤华中教授、张平文教授
筹备组组长： 樊玉伟
筹备组成员： 樊玉伟、李蔚明、李　君、马　睿、吴开亮

论坛日程

08:20-08:50 注册

08:50-09:00 开幕式

09:00-09:30 郑晖
在拟一致网格或局部加密网格上的间断系数泊松方程的多重网格算法

09:30-10:00 郭朕臣
Robust pole assignment problem via Schur form

10:00-10:30 茶歇 & 会议照相

10:30-11:00 蔡永强
A Landau model of a kind of one-dimensional quasi-periodic structure

11:00-11:30 樊玉伟
On the hyperbolicity of Grad's moment system

11:30-12:00 吴开亮
Finite volume local evolution Galerkin method for two-dimensional relativistic hydrodynamics

12:00-14:00 午餐 (中关新园和园餐厅)

14:00-14:30 胡凯博
Pointwise divergence-free and energy stable finite element methods for Navier-Stokes and incompressible MHD equations

14:30-15:00 梁鑫
Variational characterizations of eigenvalues of positive semi-definite pencils

15:00-15:30 施德才
A polynomial eigenproblem approach for general joint block diagonalization

15:50-16:00 茶歇

16:00-16:30 徐芳芳
High dimensional covariance matrix estimation using multi-factor models from incomplete information

16:30-17:00 王闻蔚
Algorithms for solving an optimization problem in CCA

17:00-17:30 吕唐杰
Photoacoustic tomography with variable sound speed

17:30-19:00 晚餐 (中关新园和园餐厅)

Conference Schedule

08:20-08:50 Registration

08:50-09:00 Opening Ceremony

09:00-09:30 Hui Zheng
在拟一致网格或局部加密网格上的间断系数泊松方程的多重网格算法

09:30-10:00 Zhenchen Guo
Robust pole assignment problem via Schur form

10:00-10:30 Coffee Break & Conference Photography

10:30-11:00 Yongqiang Cai
A Landau model of a kind of one-dimensional quasi-periodic structure

11:00-11:30 Yuwei Fan
On the hyperbolicity of Grad's moment system

11:30-12:00 Kailiang Wu
Finite volume local evolution Galerkin method for two-dimensional relativistic hydrodynamics

12:00-14:00 Lunch (Heyuan, Zhongguanyuan Global Village, PKU)

14:00-14:30 Kaibo Hu
Pointwise divergence-free and energy stable finite element methods for Navier-Stokes and incompressible MHD equations

14:30-15:00 Xin Liang
Variational characterizations of eigenvalues of positive semi-definite pencils

15:00-15:30 Decai Shi
A polynomial eigenproblem approach for general joint block diagonalization

15:50-16:00 Coffee Break

16:00-16:30 Fangfang Xu
High dimensional covariance matrix estimation using multi-factor models from incomplete information

16:30-17:00 Wenwei Wang
Algorithms for solving an optimization problem in CCA

17:00-17:30 Tangjie Lv
Photoacoustic tomography with variable sound speed

17:30-19:00 Dinner (Heyuan, Zhongguanyuan Global Village, PKU)

在拟一致网格或局部加密网格上的间断系数泊松方程的多重网格算法

郑晖
Email: zhhui@pku.edu.cn

个人简介： 郑晖，现就读于北京大学数学科学学院，2005年获得北京大学理学学士学位。

我们研究间断系数泊松方程的线性有限元离散的多重网格算法。首先我们研究拟一致（quasi-uniform）网格上的算法。直接用一般的多重网格算法求解， 收敛速度会与系数的间断程度或网格尺寸有关。我们在分解整个有限元空间的过程中发现，一些特殊的一维子空间是导致收敛速度变慢的原因。于是我们从子空间校正的角度来改进多重网格 算法，在这些特殊的一维子空间上进行额外的校正。这样，我们改进后的算法额外增加的计算量是非常小的；并且我们可以证明这种改进后的多重网格算法的一致收敛性： 即收敛速度与系数的间断程度以及网格尺寸都无关。同样地，如果我们在并行子空间校正的框架下在这些特殊的一维子空间上进行额外的校正，也可以得到一致收敛的算法。

现在我们研究局部加密的自适应网格上的多重网格算法。对于一般的泊松方程（非间断系数），我们采用只在每一层的新节点以及与之直接相邻的点处做子空间校正的多重网格算法。为了 证明这种算法的收敛性，我们提出了一种局部的L^2平均投影；每相邻两层的这种投影相减，恰好可以消去除了新节点以及与之直接相邻的点之外的所有点处的值。于是，利用这种投影 以及子空间校正的理论框架，我们可以证明这种多重网格算法的一致收敛性。对于局部加密的自适应网格上的间断系数泊松方程，我们利用以上分析非间断系数问题时的方法，以及利用我们在 拟一致网格上提出的特殊子空间，以期望得到相应的收敛性结果。

Robust Pole Assignment Problem via Schur form

Zhen-Chen Guo
Email: guozhch06@gmail.com

Biography:

Arising in the theory of control of linear dynamic systems, pole assignment problem is a special inverse algebraic eigenvalue problem, which determines certain aspects of a matrix so that the resulting matrix has the specified eigenvalues. Let the matrix pair (A, B) denotes the time invariant, linear system with dynamic state equation

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{2.1}$$

with $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$ being the open-loop system matrix and the input matrix respectively. For the controlling of the (A, B) system, one approach is to choose an input which is a linear function of the current state of the system, i.e.

$$u(t) = Fx(t) \tag{2.2}$$

for some feedback matrix $F \in \mathbb{R}^{m\times n}$. Such control is referred to as a state feedback control and the corresponding matrix

$$A_c = A + BF \tag{2.3}$$

as the closed-loop system matrix.

In applications, in order to change the dynamic behavior of the system (2.1) in some desirable way (to achieve stability or to speed up response), we need to decide a real feedback F so that A_c has a specified self-conjugate set of eigenvalues. Moreover, one desirable character for system design is that the eigenvalues of the closed-loop system matrix A_c should be insensitive to its own perturbations. Consequently, it leads to the *state-feedback robust pole assignment problem*:
State-Feedback Robust Pole Assignment Problem (SFRPA) Given $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ and a set with n complex numbers $\mathfrak{L} = \{\lambda_1, \lambda_2, \dots, \lambda_n\}$, which is closed under complex conjugation and satisfies $\lambda(A) \cap \mathfrak{L} = \emptyset$. Find $F \in \mathbb{R}^{m\times n}$ such that $\lambda(A_c) = \{\lambda_1, \lambda_2, \dots, \lambda_n\}$ and closed-loop system is robust, i.e. the eigenvalues of A_c are as insensitive to perturbations in A_c as possible.

Gourishankar and Ramar made a early contribution to the **SFRPA** problem and since then many optimization methods have been proposed based on different measurements. And almost algorithms are iterative. We propose a new direct method based on minimizing the departure from normality of the closed-loop system matrix A_c to get the real Schur form of A_c for any arbitrary set of poles. In our algorithm, we just only need to solve standard eigen-problems for both real and complex poles, which makes our algorithm much efficient. Moreover, since orthogonal matrices are used, our method is stable. And numerical experiments shows the superiorities of our algorithm.

A Landau Model of A Kind of One-Dimensional Quasi-periodic structure

Yongqiang Cai
Email: caiyq.math@gmail.com

个人简介: 蔡永强，现就读于北京大学数学科学学院， 2010年获得中南大学理学学士学位， 2013年获得中南大学理学硕士学位。

Quasicrystal is a kind of peculiar material structure which has quasiperiodic order. And it is still an unsolved problem that how quasicrystal can be stable. Our work is modeling the thermodynamic stability of some quasiperiodic structure. We focus on a kind of simple quasiperiodic structure,whose order parameter function $u(x)$ only has 4 spectral points, according to the demand of discrete spectrum of Quasicrystal:

$$u(x) = C_1 \cos x + C_2 \cos qx \tag{3.1}$$

where q is irrational, and $C_1C_2 \neq 0$.

Under the framework of Landau phenomenological theory, we modeled it's stability using the ideas proposed by Lifshitz and Petrich in 1997(LP model). The energy functional of LP model is

$$F[u] = \int\int dxdy \left\{ -\frac{1}{2}\varepsilon u^2 + \frac{c}{2}\left[\left(\nabla^2+1\right)\left(\nabla^2+q^2\right)u\right]^2 - \frac{1}{3}\alpha u^3 + \frac{1}{4}u^4 \right\} \tag{3.2}$$

where $u(x, y)$ is the order parameter function,$q, c > 0, \varepsilon, \alpha$ are parameters, and the meaning of the integrate is the average value in R^2. To analysis this model,we can use single-wave approximate method when $c \to \infty$, and numerical simulation method when c is finite.

For our 1D Quasiperiodic structure, we can use the energy functional like LP model:

$$F[u] = \frac{c}{2}\int \left[\left(\nabla^2+1\right)\left(\nabla^2+q^2\right)u\right]^2 dx + \int f(u)dx \tag{3.3}$$

where f is any polynomial whose highest term has positive coefficient. Our purpose is find some f such that the minimum of $F[u]$ can only be got at some quasiperiodic functions, when $c \to \infty$. But our first result points out the impossibility of this aim, which means that the energy functional should be modified. Adding some terms (mathematically) on the functional (3.3), we can achieve that goal. And the stability condition of our 1D Quasiperiodic structure and the phase transitions are given.

Single-wave approximate method is to deal with the ideal case of the c approaching to infinite. In the other case, for finite positive c, we employ the numerical simulation method. In order to improve the numerical efficiency, we employ the higher dimensional projection simulation method, used by Kai Jiang firstly, instead of the classical periodical approximation methods. This method has been constructed according to the fact that Quasiperiodic structure can be seen as a projected higher dimensional periodic crystal. It only requests a higher dimensional rectangular area to numerical calculation but the whole 1D space.

Comparing the results of the single-wave approximation method and the numerical simulation method, we find that single-wave approximate not only can be used to the extreme situation of parameters c tend to infinite, but also to assist the normal situation to get the solutions and phase diagrams.

References

[1] Lifshitz Ron,Petrich Dean M., Theoretical Model for Faraday Waves with Multiple-Frequency Forcing. Physical Review Letters, 1997. 79(7): p. 1261-1264.

[2] Kai Jiang,Pingwen Zhang, Numerical Methods for Quasicrystals. arXiv preprint arXiv:1212.3326, 2012.

On the hyperbolicity of Grad's moment system

Yuwei Fan
Email: dawanzhi@163.com

Biography: Yuwei Fan obtained Bachelor Degree in Science, Peking University in 2011, and has been Master candidate from 2011 to 2013 in School of Mathematical Sciences, Peking University, and now he is Ph. D. candidate in the same department. His supervisor is Prof. Ruo Li. He was awarded the first prize of Sixth Youth Outstanding Paper Prize, which is presented by Chinese Computational Mathematical Society in 2013. His research interests include model and numerical methods for rarefied gas dynamics and microflow.

Nowadays, the kinetic gas theory is drawing increasing attentions in the high-tech fields. The kinetic theory is considered as a mesoscopic description of fluids, which is based on the classical Boltzmann equation from statistical physics. However, a full accurate mesoscopic model is still too complex for lots of problems. During a long period of time, people have been looking for a median model between the classical macroscopic equations and the Boltzmann equation. This can be tracked back to the work of Burnett [2]. As is well known, the Burnett equations are later proved to be linearly unstable by Bobylev [1]. Another way leading to linearly stable intermediate models is the moment method proposed by Grad's [7]. Since this method was discarded by Grad's himself, very few works contributed to this area in the last century. However, this field is becoming active in the recent years, since people find that some traditional difficulties in the moment equations can be ignored by some regularizations to these models.

This talk focuses on a major critique of the moment method — the lack of global hyperbolicity for Grad's moment system. This deficiency directly causes blow-ups when the distribution is not in the hyperbolic region. Grad's 13 moment system, most well-known system of Grad's moment system, attacks a lot of attentions of researchers. I. Muller [8] points out it is not globally hyperbolic and for 1D flow, the hyperbolicity can only be obtained near the equilibrium. Several extensions of this model are proposed. However, all the analysis on the hyperbolicity is restricted to 1D flows, while there are no comments indicating that the 3D case is the same or similar as the 1D case. In this talk, we will point out that in the 3D Grad's 13 moment system, for each equilibrium state, none of its neighbourhoods is contained in the hyperbolicity region [6]. This reveals that an arbitrary small perturbation of the equilibrium state may lead to the loss of hyperbolicity.

A worse news is that increasing the number of moments shows no improvements in numerical experiments [5]. Hence, proposing a hyperbolic regularization is urgent and significant. In this talk, we will proposed a regularization for arbitrary order of Grad's moment system to achieve globally hyperbolicity [3, 4].

References

[1] A. V. Bobylev. The Chapman-Enskog and Grad methods for solving the Boltzmann equation. *Sov. Phys. Dokl.*, 27(1):29–31, 1982.

[2] D. Burnett. The distribution of velocities in a slightly non-uniform gas. *Proc. London Math. Soc.*, 39(1):385–430, 1935.

[3] Z. Cai, Y. Fan, and R. Li. Globally hyperbolic regularization of Grad's moment system. *Comm. Pure Appl. Math. (online)*, 2013.

[4] Z. Cai, Y. Fan, and R. Li. Globally hyperbolic regularization of Grad's moment system in one dimensional space. *Comm. Math Sci.*, 11(2):547–571, 2013.

[5] Z. Cai, R. Li, and Y. Wang. Numerical regularized moment method for high Mach number flow. *Commun. Comput. Phys.*, 11(5):1415–1438, 2012.

[6] Z.-N. Cai, Y.-W. Fan, and R. Li. On hyperbolicity of 13-moment system. *Tech Report, Institude of Math, Peking Univeristy*, 2013.

[7] H. Grad. On the kinetic theory of rarefied gases. *Comm. Pure Appl. Math.*, 2(4):331–407, 1949.

[8] I. Müller and T. Ruggeri. *Rational Extended Thermodynamics, Second Edition*, volume 37 of *Springer tracts in natural philosophy.* Springer-Verlag, New York, 1998.

Finite volume local evolution Galerkin method for two-dimensional relativistic hydrodynamics

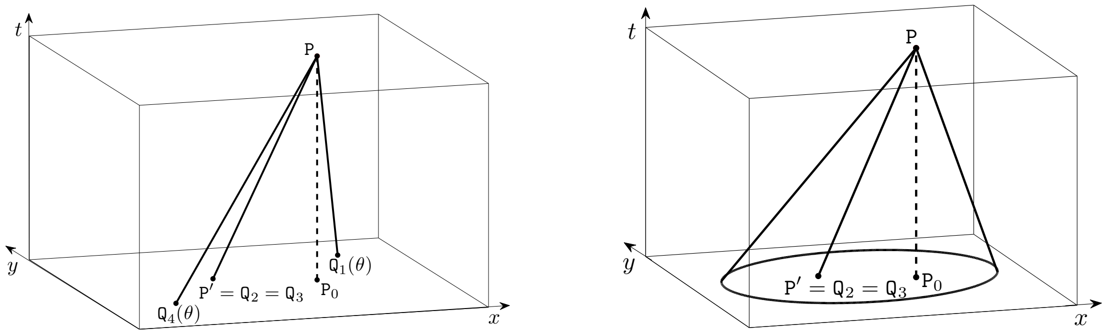
Kailiang Wu
Email: wukl@pku.edu.cn

Biography: Kailiang Wu received his B.E. Degree in School of mathematics and statistics from Huazhong University of Science and Technology, Wuhan, China, in 2011. He is now a Ph.D. candidate in the School of Mathematical Sciences at Peking University. He has published 2 papers indexed in the SCI during these two years. His research interests include numerical methods for the hyperbolic conservation laws, relativistic numerical hydrodynamics, generalized Riemann problem, Vlasov systems, nonlinear Dirac model, and adaptive mesh method.

Relativistic hydrodynamics (RHD) plays a major rule in many fields of modern physics, e.g. astrophysics, high-energy and nuclear physics, etc. Relativistic flows appear in numerous astrophysical phenomena from stellar to galactic scales, e.g. active galactic nuclei, super-luminal jets, core collapse super-novae, X-ray binaries, pulsars, coalescing neutron stars and black holes, micro-quasars, and gamma-ray bursts, etc. The relativistic description of fluid dynamics should be taken into account if the local velocity of the flow is close to the light speed in vacuum or the matter is influenced by large gravitational potentials, where the Einstein field theory of gravity has to be considered.

The dynamics of the relativistic systems requires solving highly nonlinear equations and the analytic treatment of practical problems is extremely difficult. Hence, numerical simulation is one of the indispensably important approaches for their studies. Wilson first attempted to solve the general RHD equations in an Eulerian form with an explicit finite difference code [4], which is combined with the artificial viscosity techniques to handle the discontinuous solutions. After that, various shock-capturing methods have been developed for the RHD equations. Most of those methods for the multi-dimensional RHD equations are based on the 1D (exact or approximate) Riemann solver. Unfortunately, such multi-dimensional methods, based on 1D (exact or approximate) Riemann solver, may lead to a misinterpretation of local wave structure in the solutions of the quasi-linear hyperbolic system [2]. For example, when waves are propagating in directions that are oblique with respect to the mesh lines, the local Riemann solver methods may lead to structural deficiencies and large errors in the numerical solutions [1, 3]. As it was validated in the analytic derivation of the direct Eulerian GRP scheme in [6], the multi-dimensional effect in the 2D RHD equations is very strong due to the appearance of the Lorentz factor. Thus it is interesting to develop genuinely multi-dimensional methods for the multi-dimensional RHD equations.

In the talk, I will introduce the genuinely multi-dimensional finite volume local evolution Galerkin (FVLEG) method for two-dimensional special RHD equations [5]. Instead of using the dimensional splitting method or solving one-dimensional local Riemann problem in the direction normal to cell interface, the FVLEG method couples a finite volume formulation with the genuinely multi-dimensional approximate local evolution operator, which is derived by evolving the solutions of corresponding locally linearized RHD equations along all bicharacteristic directions, see Fig. 5.1. The derivation of the evolution operator for the 2D RHD system is much more difficult than that of the non-relativistic Euler equations [1] due to its complex characteristic structures and the implicit nonlinear relations from the conservative variables and the primitive variables. Several numerical results will be shown to demonstrate the accuracy and the



(a) (b)

Figure 5.1: The bicharacteristic curves with the fixed angle θ (left) and the bicharacteristic cone (right) of the locally linearized 2D RHD system.

performance of the proposed FVLEG method.

References

[1] M. Lukáčová-Medvid'ová, K.W. Morton, and G. Warnecke, Finite volume evolution Galerkin methods for Euler equations of gas dynamics, *Int. J. Numer. Meth. Fluids*, 40(2002), 425-434.

[2] P. Roe, Discrete models for the numerical analysis of time-dependent multidimensional gas dynamics, *J. Comput. Phys.*, 63(1986), 458-476.

[3] Y.T Sun and Y.X. Ren, The finite volume local evolution Galerkin method for solving the hyperbolic conservation laws, *J. Comput. Phys.*, 228(2009), 4945-4960.

[4] J.R. Wilson, Numerical study of fluid flow in a Kerr space, *Astrophys.J.*, 173(1972), 431-438.

[5] K.L. Wu and H.Z. Tang, Finite volume local evolution Galerkin method for two-dimensional relativistic hydrodynamics, *J. Comput. Phys.*, 256(2014), 277-307.

[6] Z.C. Yang and H.Z. Tang, A direct Eulerian GRP scheme for relativistic hydrodynamics: two-dimensional case, *J. Comput. Phys.*, 231(2012), 2116-2139.

Pointwise Divergence-free and Energy Stable Finite Element Methods for Navier-Stokes and Incompressible MHD Equations

Kaibo Hu
Email: hukaibo02@gmail.com

个人简介: 胡凯博，2012级直博生。方向为有限元方法，保结构算法，具体背景包括MHD方程，Einstein方程等

In this talk, we will first review basic ideas of Finite Element Exterior Calculus (FEEC), and utilize this powerful tool to give two strong divergence-free and energy stable finite element discretizations for Navier-Stokes and MHD equations. Some possible connections to Einstein field equations and numerical relativity will also be mentioned.

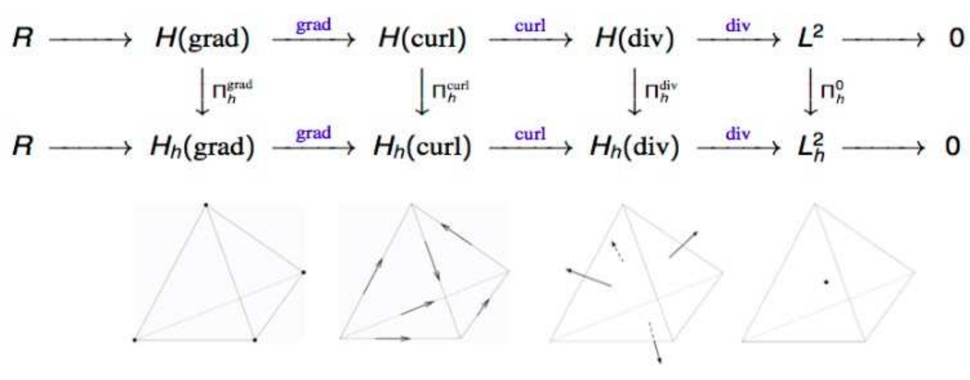
Face and edge elements are two successful families of elements for $H(\text{div})$ and $H(\text{curl})$ spaces. Using the language of differential forms, people realized that they coincide with the "Whitney form" in algebraic topology. Hence a canonical construction gives the finite element periodic table, including Lagrange, Raviart-Thomas, Nédélec, BDM, DG etc. elements of arbitrary order in any dimension.

Numerical stability of saddle point systems crucially depends on the differential structure preserved from continuous setting. With the above compatible spatial discretization, discrete inf-sup condition is naturally satisfied, which leads to the stability.

In the second part, we will present two structure-preserving finite element discretizations for MHD equations, in which $u \in H_{0,h}(\text{div})$ and $B \in H_h(\text{div})$ are both pointwise divergence-free. Continuous energy estimate is preserved on the discrete level.

$$\begin{aligned}&\frac{1}{2\Delta t}(\|B^{n+1}\|^2 - \|B^n\|^2) + \frac{\mu}{2\mu_0 \Delta t}(\|u^{n+1}\|^2 - \|u^n\|^2)\\ &\quad = -\frac{1}{\mu\sigma}(\nabla_h \times B^{n+\frac{1}{2}}, \nabla_h \times B^{n+\frac{1}{2}}) - \frac{\mu}{\mu_0}\nu(\nabla_h \times u^{n+\frac{1}{2}}, \nabla_h \times u^{n+\frac{1}{2}})\end{aligned}$$

Finite element de Rham sequence and discrete Hodge decomposition are the main tools in the new method. Similar to continuous system, pointwise divergence-free of u plays a crucial role in the energy estimates. This method is a generalization of the classical MAC scheme to finite element methods in triangular mesh in the sense of mass lumping.



Similar idea can also be applied to Method of Characteristic (MOC). MOC uses Lagrange formulation to discretize material derivative in Navier-Stokes and MHD equations, and can be considered as a generalization of up-winding schemes. Pointwise divergence-free and discrete energy estimate also hold.

$$
\begin{aligned}
&\frac{\mu}{\mu_0}\frac{1}{\Delta t_1}\frac{1}{2}(\|u^{n+1}\|^2 - \|u^n\|^2) + \frac{1}{\Delta t_3}(\|B^{n+1}\|^2 - \|B^n\|^2) \\
&\qquad \leq -\frac{\mu\nu}{\mu_0}(\nabla_h \times u^{n+1}, \nabla_h \times u^{n+1}) - \frac{1}{\mu\sigma}(\nabla_h \times B^{n+\frac{1}{2}}, \nabla_h \times B^{n+\frac{1}{2}})
\end{aligned}
$$

In order to get this energy estimate, we use Lie derivative formulation to rewrite Lorentz form in weak form of momentum equation, and eliminate with induction equation in discrete setting, as one does in continuous system.

In this Langrangian case, volume-preserving of flow mapping is crucial for energy property, which is an integration version of divergence-free condition. This can be got by symplectic methods in 2 dimension, and with some modifications in higher dimension.

Variational Characterizations of Eigenvalues of Positive Semi-Definite Pencils

Xin Liang
Email: liangxinslm@pku.edu.cn

个人简介: 梁鑫，现就读于北京大学数学科学学院，2009年获得北京大学理学学士学位。 2009-2011年度获得北京大学校长奖学金。发表SCI期刊论文2篇，其中一篇被引用1次。

This talk will be a summary of [6] and [5]. This work is concerned with variational characterizations of eigenvalues of positive semi-definite pencils.

We call a matrix pencil $A - \lambda B$ is positive (semi-)definite if A, B are Hermitian and there exists $\lambda_0 \in \mathbb{R}$ such that the matrix $A - \lambda_0 B$ is positive (semi-)definite.

This research begins with the trace minimization principle of the positive semi-definite pencils.

In 1995, Kovač-Striko and Veselić [3] told many results the main part of which is: Let $A - \lambda B$ be a positive semi-definite pencil where the inertia of B is $(n_+, 0, n_-)$, and denote its eigenvalues by $\lambda^-_{n_-} \le \cdots \le \lambda^-_1 \le \lambda^+_1 \le \cdots \le \lambda^+_{n_+}$, then

$$\inf_{X^{\mathrm{H}} BX = J_k} trace(X^{\mathrm{H}} AX) = \sum_{i=1}^{k_+} \lambda^+_i - \sum_{i=1}^{k_-} \lambda^-_i, \tag{7.1}$$

where $J_k = diag(I_{k_+}, -I_{k_-})$. Note that the assumption on the inertia of B implies that B is nonsingular and $A - \lambda B$ is a regular pencil.

In that paper, they suspected their result would be true without this assumption. Our first contribution is that (7.1) holds indeed when B is no longer assumed nonsingular, and we even go further to allow $A - \lambda B$ to be a singular pencil.

As a by-product, we give out a canonical representation of this type of pencils, and then we obtained a sufficient necessary condition for the attainability of the infimum in (7.1) in terms of certain indices in the canonical representation.

Other variational characterizations of eigenvalues of positive semi-definite pencils have been studied for some time.

Extensions of Courant-Fischer principles are available for a regular Hermitian pencil with nonsingular B [2, 4, 7, 10] and with possibly singular B [1, 8]. In the extensions, only certain real semi-simple eigenvalues admit a characterization.

Nakić and Veselić [9] obtained an extension of Wielandt's principles for a regular Hermitian pencil with nonsingular B and whose *none-cancelled* real eigenvalues are all semi-simple[1].

In our work, we extended these principles too, as in [9], but to a positive semi-definite Hermitian pencil with B possibly singular and as well as $A - \lambda B$ being possibly singular.

Indeed, we have the principles which are more general: for any function which is non-decreasing in each of its arguments, the sup-inf and inf-sup optimizations are respectively the function values of some eigenvalues of the matrix pencil.

[1] The reader is cautioned about a few inaccurate/incorrect statements/equations in [9], however.

Acknowledgement

This work was supported in part by China Scholarship Council. This work was mainly done when this author was a visiting student to Department of Mathematics, University of Texas at Arlington, Arlington, TX 76019, USA. The author sincerely acknowledges Dr. Ren-Cang Li[2] , who played an important role in this work as a co-author and advisor.

References

[1] Paul Binding, Branko Najman, and Qiang Ye. A variational principle for eigenvalues of pencils of Hermitian matrices. *Integr. Eq. Oper. Theory*, 35:398–422, 1999.

[2] Paul Binding and Qiang Ye. Variational principles for indefinite eigenvalue problems. *Linear Algebra Appl.*, 218:251–262, 1995.

[3] J. Kovač-Striko and K. Veselić. Trace minimization and definiteness of symmetric pencils. *Linear Algebra Appl.*, 216:139–158, 1995.

[4] P. Lancaster and Q. Ye. Variational properties and Rayleigh quotient algorithms for symmetric matrix pencils. *Oper. Theory: Adv. Appl.*, 40:247–278, 1989.

[5] Xin Liang and Ren-Cang Li. Extensions of Wielandt's min-max principles for positive semi-definite pencils. *Linear Multilinear Algebra*, 2013. DOI:10.1080/03081087.2013.803242.

[6] Xin Liang, Ren-Cang Li, and Zhaojun Bai. Trace minimization principles for positive semi-definite pencils. *Linear Algebra Appl.*, 438:3085–3106, 2013.

[7] B. Najman and Q. Ye. A minimax characterization of eigenvalues of Hermitian pencils. *Linear Algebra Appl.*, 144:217–230, 1991.

[8] B. Najman and Q. Ye. A minimax characterization of eigenvalues of Hermitian pencils II. *Linear Algebra Appl.*, 191:183–197, 1993.

[9] I. Nakić and K. Veselić. Wielandt and Ky-Fan theorem for matrix pairs. *Linear Algebra Appl.*, 369:77–93, 2003.

[10] Qiang Ye. *Variational principles and numerical algorithms for symmetric matrix pencils.* PhD thesis, University of Calgary, Calgary, Canada, 1989.

[2] Department of Mathematics, University of Texas at Arlington, P.O. Box 19408, Arlington, TX 76019. E-mail: rcli@uta.edu. Supported in part by NSF grants DMS-0810506, DMS-1115834 and DMS-1317330, and a Research Gift Grant from Intel Corporation.

(General) Joint Block Diagonalization of Hermitian matrix set

Decai Shi
Email: decaishi@gmail.com

个人简介: 个人简介: 施德才,现就读于北京大学数学科学学院, 2007年获得南京理工大学理学学士学位, 2009年获得南京理工大学理学硕士学位。

This talk will be about the General Joint Block Diagonalization of Hermitian matrix set.

The problem of Joint Block Diagonalization(JBD) is stated as follows. Given a matrix set $\mathcal{A} = \{A_i\}_{i=0}^p$ with A_i Hermitian matrix of order n, and a partition $\tau_n = (n_1, n_2, \ldots, n_t)$, JBD aims to find a nonsingular matrix $W = W(\tau_n)$ such that

$$W^H A_i W = \mathrm{d}iag(A_{i1}, A_{i2}, \ldots, A_{it}), \quad \text{for} \quad i = 0, 1, 2, \ldots, p, \tag{8.1}$$

where $A_{ij} \in \mathbb{C}^{n_j \times n_j}$ for $j = 1, 2, \ldots, t$.

General JBD (GJBD) attempts to solve JBD without knowing the block diagonal structure, i.e., given a matrix set $\mathcal{A} = \{A_i\}_{i=0}^p$ with A_i Hermitian matrix of order n, GJBD aims to find a partition $\tau_n' = (n_1', n_2', \ldots, n_t')$ and a nonsingular matrix $W = W(\tau_n')$ such that

$$card(\tau_n') = \max\{card(\tau_n) \,\big|\, \text{there exists a nonsingular matrix } W = W(\tau_n) \text{ solves NU-JBD.}\}$$

GJBD arises in Independent Subspace Analysis (ISA)/Blind Source Separation (BSS), and is more difficult than JBD.

In this talk, we show that JBD and GJBD of $\{A_i\}_{i=0}^p$ are strongly connected with the eigeninformation of the associated matrix polynomial $P_{\mathcal{A}}(\lambda) = \sum_{i=0}^p \lambda^i A_i$. Our discussion is carried out under the assumption that the self-adjoint matrix polynomial $P_{\mathcal{A}}(\lambda)$ has only simple eigenvalues.

First, two necessary and sufficient conditions for JBD are given. One is characterized by the pn eigenvectors of $P_{\mathcal{A}}(\lambda)$ and the other is characterized by the set $\mathbb{S}_{(X,\Lambda)}$ which is closely related to the standard pair (X, Λ) of $P_{\mathcal{A}}(\lambda)$.

Second, by use of the necessary and sufficient condition for JBD and the structure of elements in $\mathbb{S}_{(X,\Lambda)}$, a solvability theory for GJBD is established and the solutions of GJBD are characterized by the eigeninformation. What's more, we have $card(\tau_n') = \zeta_n^{opt}$, which is the eigenvalue multiplicity vector for $\mathbb{S}_{(X,\Lambda)}$. Specially, we have $card(\tau_n') = \dim(\mathbb{S}_{(X,\Lambda)})$ when the leading coefficient matrix A_p is positive definite.

Based on the established theory, a numerical method is proposed to solve GJBD. Numerical tests show that this method is not only feasible for exact GJBD, but also able to handle approximate GJBD to certain extend.

High Dimensional Covariance Matrix Estimation Using Multi-factor Models from Incomplete Information

Fangfang Xu
Email: happyxufangfang@126.com

个人简介: 徐芳芳，现就读于上海交通大学数学系。她曾获得孙越崎优秀学生奖，全国研究生数学建模大赛二等奖。参与合作论文5篇。

Covariance matrix estimation plays a fundamental role in multivariate analysis and has a wide range of applications, such as risk management, asset pricing and portfolio allocation. Traditionally, covariance matrix is estimated by sample covariance matrix. It is an unbiased estimator and the sample covariance matrix is invertible when the dimensionality is much smaller than the sample size. However, this task becomes very challenging in the high dimensional setting where the dimensionality is comparable or much larger than the sample size. In this situation, the sample covariance matrix is singular. Even if the sample covariance matrix is invertible, the expected value of its inverse is a biased estimator for the theoretical inverse. Therefore, it is hard or even unrealistic to estimate covariance matrices well without imposing any structure in this case. A widely used approach for reducing dimensionality is based on multi-factor models. Although it has been well studied and quite successful in many applications, the quality of the estimated covariance matrix is often degraded due to a nontrivial amount of missing data in the factor matrix for both technical and cost reasons. Since the factor matrix is only approximately low rank or even has full rank, existing matrix completion algorithms are not applicable.

We aim to complete the factor matrix and estimate high dimensional covariance matrix from incomplete information under the condition that the factor matrix is approximately low-rank or even has full rank. Our main contribution is the development of efficient algorithms for this task. Firstly, three models based on a multi-factor model are proposed. The structure of them suggests an alternating minimization scheme, which is very suitable for solving large-scale problems. In this case, one can update each of the variables efficiently while fixing the others. Closed-form solution of each subproblem can be gotten and three new algorithms are present. Although the new models are non-convex, we prove that our algorithms converge to their stationary points, respectively. We test our proposed algorithms on two kinds of problems: noiseless matrix problems and real data with noise. Numerical experiments show that our proposed algorithms output satisfactory results.

Reference: [1] J. Fan, Y. Liao, and M. Mincheva. High-dimensional covariance matrix estimation in approximate factor models. The Annals of Statistics, 39:3320-3356, 2011. [2] T. T. Cai and M. Yuan. Adaptive covariance matrix estimation through block thresholding. The Annals of Statistics, 40(4):2014-2042, 2012. [3] E. J. Candes and B. Recht. Exact matrix completion via convex optimization. Foundations of Compu- tational Mathematics, 9:717-772, May 2009.

Formulation and Algorithms for Sparse Canonical Correlation Analysis

Wenwei Wang
Email: pkuwww89@gmail.com

Biography: Wenwei Wang received her B.S. Degree in School of Mathematical Sciences, Peking University in 2011. She is now a master candidate in the Department of Scientific & Engineering Computing, SMS, PKU and major in Optimization.

Canonical Correlation Analysis(CCA),which was first proposed by H. Hotelling, is a classical method uesd to find the correlation between two sets of variables in multivariate data analysis. It first seeks the pair of linear transformations associated to the two sets such that the projected variables are maximally correlated and then seeks the pair of linear transformations resulting in the largest correlation among all pairs whose projected variables are uncorrelated with the selected pair, and so on. CCA has been widely applied to economics, meteorology, geology, etc.

Let $x \in \mathbb{R}^{d_1}$ and $y \in \mathbb{R}^{d_2}$ be two random variables. $\Sigma_{xx} \in \mathbb{R}^{d_1 \times d_1}$, $\Sigma_{yy} \in \mathbb{R}^{d_2 \times d_2}$ are covariance matrices for variables x and y, respectively, and $\Sigma_{xy} \in \mathbb{R}^{d_1 \times d_2}$ is the cross-covariance matrix between x and y. CCA computes the pair of $w_x \in \mathbb{R}^{d_1}$ and $w_y \in \mathbb{R}^{d_2}$ as the optimal solution of the following optimization problem

$$\begin{array}{rl} \max & w_x^\top \Sigma_{xy} w_y \\ s.t. & w_x^\top \Sigma_{xx} w_x = 1,\ w_y^\top \Sigma_{yy} w_y = 1. \end{array} \tag{10.1}$$

Traditionally, the optimal solution is obtained by solving an generalized eigenvalue problem, which is under the assumption that Σ_{yy} is nonsingular. However, for high dimensional data, especially when the number of samples is smaller than the data dimension, both Σ_{xx} and Σ_{yy} are singular, which makes it challenging to solve the problem above. What's more, there may be many solutions to (10.1), and we would like to choose the optimal ones according to some added criteria.

In practical applications, it is very important to interpret the canonical variables in high dimensional data analysis. In fact, many of the features are not valuable to the interpretation, so we requires the canonical loadings w_x and w_y to be sparse, and derive a modified CCA problem, namely sparse Canonical Correlation Analysis(SCCA). Recently, SCCA has been studied by a few researchers. The main difficulty of this problem is that the feasible region is nonconvex, thus it is hard to apply classical optimization algorithms directly. D. R. Hardoon et al.(2007) replaced the original constraint with $\|y\|_\infty = 1$ and proposed a novel algorithm, while D. M. Witten(2009) replaced it with $\|w_x\|_2 = \|w_y\|_2 = 1$ and solved the new problem by a penalized matrix decomposition. However, there is few theoretical basis for such replacement and sometimes we cannot obtain satisfactory rates of sparsity. We formulate the SCCA problem without changing the original constraints and propose a new method based on an alternating direction scheme. In each step, the corresponding subproblem can be solved efficiently. Numerical experiments show the effectiveness of our method.

Photoacoustic Tomography with Variable Sound Speed

Tangjie Lv
Email: lvtangjie690@163.com

个人简介: 吕唐杰，现就读于北京大学数学科学学院， 2009年获得北京大学理学学士学位。 曾获北京大学“中国电科十四所国睿”奖学金、北京大学“五四”奖学金。

Photoacoustic Tomography or Thermoacoustic Tomography is a developing medical imaging method in recent decades. This is a hybrid medical imaging method charaterized by high revolution and constrast. There are various types of reconstruction methods to solve this problem when the sound speed is constant. However, they will not be efficient for variable sound speed. Several image reconstruction methods have been proposed when the sound speed is variable, but the reconstructions are not so well. We investigate the adjoint equation in photoacoustic tomography with variable sound speed, and propose three variational iterative algorithms. The basic idea of these algorithms is to compute the original equation and the adjoint equation iteratively. We present numerical examples and show the well performance of these variational iterative algorithms.